2. The Problem of Quantum Statistics

In classical nineteenth century atomic physics the strongest version of PII with attributes restricted to monadic intrinsic properties, is clearly false for the atoms. ~~and molecules~~. The weak version is however true if we allow an impenetrability assumption (IA) to the effect that two ~~atoms~~ distinct atoms can never occupy the same location in space. We want to begin our discussion of quantum physics by considering an argument to the effect that quantum particles cannot be ~~individuated at all~~ regarded as individuals at all. ~~Hence the problem~~ If this were the case, the problem of how they are individuated simply would not arise. PII would be not be either true or false but simply ~~is ap~~ inapplicable.

The argument runs like this. Consider the problem of distributing two quantum particles# initially supposed to be individuals‡ and labelled 1 and 2, among two possible pure quantum states $|a^r\rangle$ and $|a^s\rangle$ which we may suppose take eigenstates of some maximal observable A for each particle with eigenvalues a^r and a^s as indicated by the notation for the states‡. By analogy with the situation in classical physics we might suppose that there are four possibilities:

(1) Both particles are in the state $|a^{n}\rangle$
(2) Both particles are in the state $|a^{s}\rangle$
3) Particle 1 is in state $|a^{n}\rangle$ and particle 2 in state $|a^{s}\rangle$
(4) Particle 1 is in state $|a^{s}\rangle$ and particle 2 in state $|a^{n}\rangle$.

~~All these four possible arrangements would be given equal weight in classical statistical mechanics~~

~~But in quantum statistics things are different~~

Now in classical statistical mechanics arrangements 3 and 4 would be counted as distinct and given equal weight in assigning probabilities. But in quantum statistics, whether bosonic or fermionic, the arrangements 3 and 4 are counted as one and the same arrangement for the purposes of assigning weights. This is taken to show that the two arrangements are not only indistinguishable but are actually identical. But ontologically speaking these two arrangements are not identical if the two quantum particles are individuals. Hence the quantum particles cannot be individuals.

In passing we may note that this argument, while purporting to show that quantum particles fall outside the scope of PII, since they are not individuals, is also sometimes invoked to show that

PII does apply to the states of affairs represented by the two arrangements 3 and 4 5.
What about arrangements 1 and 2?
~~Since the states~~ this is where bosons differ from fermions. For bosonic particles 1 and 2 are allowed arrangements to be counted with equal weight as compared with the single identified 3-cum-4 arrangement. For fermionic particles however arrangements 1 and 2 are not permitted at all. This is the famous Pauli Exclusion Principle.
We return in the next section to discuss the significance of this difference from the point of view of PII.
But first we need to explain what is wrong with the argument concerning the identity of the arrangements 3 and 4.
We begin by writing down the state vectors for the combined two-particle system corresponding to the arrangements 1, 2 3 and 4. They are:

$$|a^r\rangle \otimes |a^r\rangle \qquad (1)$$

$$|a^s\rangle \otimes |a^s\rangle \qquad (2)$$

$$|a^r\rangle \otimes |a^s\rangle \qquad (3)$$

and
$$|a^s\rangle \otimes |a^r\rangle \qquad (4)$$

where we use the convention that in a tensor product of two states the left-hand member refers to particle 1 and the right-hand member to particle 2.

Now it is quite true that if the quantum particles are individuals then the states $\overline{\mathrm{III}}^{(3)}$ and $\overline{\mathrm{IV}}^{(4)}$ are not identical. But the important point to notice is that these states are not the ones used in discussing quantum statistical mechanics. The relevant states for that purpose are as follows:

$$|a^r\rangle \otimes |a^r\rangle \quad \text{--- (5)}$$

$$|a^s\rangle \otimes |a^s\rangle \quad \text{--- (6)}$$

$$\tfrac{1}{\sqrt{2}}\left(|a^r\rangle \otimes |a^s\rangle + |a^s\rangle \otimes |a^r\rangle\right) \quad \text{--- (7)}$$

and

$$\tfrac{1}{\sqrt{2}}\left(|a^r\rangle \otimes |a^s\rangle - |a^s\rangle \otimes |a^r\rangle\right) \quad \text{--- (8)}$$

The four states $\overline{\mathrm{V}}^{(5)}$, $\overline{\mathrm{VI}}^{(6)}$, $\overline{\mathrm{VII}}^{(7)}$ and $\overline{\mathrm{VIII}}^{(8)}$ are mutually orthogonal and span the same ~~vector~~ subspace as the states $\overline{\mathrm{I}}^{(1)}$, $\overline{\mathrm{II}}^{(2)}$, $\overline{\mathrm{III}}^{(3)}$ and $\overline{\mathrm{IV}}^{(4)}$, but they are chosen so that $\overline{\mathrm{V}}^{(5)}$, $\overline{\mathrm{VI}}^{(6)}$ and $\overline{\mathrm{VII}}^{(7)}$ are symmetric under exchange of particle labels (i.e. under exchange of left-hand and right-hand members of ~~all~~ tensor products) while $\overline{\mathrm{VIII}}^{(8)}$ is antisymmetric (changes sign) under the same operation.

Note that $\overline{\mathrm{V}}^{(5)}$ and $\overline{\mathrm{VI}}^{(6)}$ are the same states as $\overline{\mathrm{I}}^{(1)}$ and $\overline{\mathrm{II}}^{(2)}$. The crucial difference is between the pairs $\overline{\mathrm{III}}^{(3)}$ and $\overline{\mathrm{IV}}^{(4)}$, and $\overline{\mathrm{VII}}^{(7)}$ and $\overline{\mathrm{VIII}}^{(8)}$.

Now $\overline{\mathrm{VII}}^{(7)}$ is no more identical with $\overline{\mathrm{VIII}}^{(8)}$ than is $\overline{\mathrm{III}}^{(3)}$ with $\overline{\mathrm{IV}}^{(4)}$.

but for bosons the states are restricted to the three symmetric possibilities. That is why VIII(8) gets eliminated from the counting procedures, not because it gets identified with VII(7)!
Similarly for fermions the states are restricted to the antisymmetric possibilities.
But in this simple example this eliminates V(5), VI(6) or VII(7), so VIII(8) alone gets counted, but again not because it gets identified with VII(7).
To put the matter another way, states with the wrong symmetry get eliminated because they are not accessible to the joint quantum system, not because ~~they do~~ there are no such states!
It should be remembered that for time-evolution under a symmetric Hamiltonian[6] the symmetry character of a state cannot change with time so no transitions can occur between of symmetric bosonic states and antisymmetric fermionic states.
The upshot ~~...~~ of our argument is to show, not that quantum particles must be 'individuals' but rather that it is POSSIBLE for them to be individuals, ~~even in the light~~ despite the peculiarities of quantum statistics.
It is quite true that in quantum field theory (QFT) particles are not regarded as individuals. They are simply (quantized) excitations of a field. Particle labels do not enter into the discussion at all.

If our simple problem of counting the number of states for a two-particle system distributed over two one-particle states were transposed to quantum field theory, then for a boson (commuting) field there would be just three states corresponding to a double excitation of ~~each~~ either state (mode) plus a single excitation of both states (modes). Similarly for a fermionic ~~field~~ (anticommuting) field, there is only one state since double excitations are not allowed. So the quantum statistics comes out the way we want it to.

It is also true that there are strong arguments for regarding the 'quantized excitation' view of quantum particles as the correct one.[7] However for the purposes of this paper we continue to envisage the possibility of treating quantum particles as individuals and ~~then~~ proceed to discuss whether they would or would not obey P.I.I.

3. The Indistinguishability Postulate

What do mean by saying that two quantum particles of the same species (characterized by their intrinsic properties) are indistinguishable?

In quantum mechanics this is expressed by the Indistinguishability Postulate (IP)

$$\langle P\phi | Q | P\phi \rangle = \langle \phi | Q | \phi \rangle \qquad (9)$$
$$\forall Q, \forall \phi$$

where ϕ is an arbitrary N-particle state and Q a possible observable on the N-fold tensor product space of states. $|P\phi\rangle$ is an abbreviation for $P|\phi\rangle$ where P is the unitary operator which is associated with an arbitrary permutation of the particle labels.

(9) says ~~effectively~~ that it is not possible to tell by measuring the expectation value of any observable, whether the state of the system is $|\phi\rangle$ or $|P\phi\rangle$

We notice that a sufficient condition for (9) to hold is that $|P\phi\rangle = \pm|\phi\rangle$ for any self-adjoint operator on the N-particle state-space.

This interprets (9) as a restriction on the possible states for the N-particle system, ~~restricting them~~ allowing just the boson or fermion possibility.

(Note that the choice of sign needs only

exist, allowing for the possibility of so-called para-statistics intermediate in character between bosonic and fermionic behaviour.

But even in the two-particle case it should be noted that the Messiah and Greenberg approach does not restrict the available states, only their accessibility in the way we described in the previous section.

From now on we shall restrict the discussion mainly to the simple two-particle case.

Let Q now denote a possible observable (self-adjoint operator) on a single particle. Considered as possible observables on the joint system we have two possibilities $Q \otimes I$ for observing Q on particle 1 and $I \otimes Q$ for observing Q on particle 2. Denote $Q \otimes I$ by Q_1 and $I \otimes Q$ by Q_2. Then PI says in the Messiah-Greenberg interpretation, that although Q_1 and Q_2 are self-adjoint operators in the Hilbert space for the joint system, they cannot actually be observed. The intuition here is that observing Q_1 or Q_2 would involve knowing empirically which particle was which and this is impossible if the particles are indistinguishable.

But from the point of view of discussing PI, it seems clear that

Insertion

* and their associated 'actualization' probabilities[8]

we should not restrict the discussion to attributes which can actually be observed. ~~If we did this then by our very definition of indistinguishability PII would automatically be violated~~ This would restrict the discussion to symmetric combinations such as $Q_1 + Q_2$. The ontological significance of PII can only be brought out by discussing whether particles 1 and 2 have the same physical attributes expressed by Q_1 and Q_2 *[number] while recognizing that these attributes can never be observed!

This is the task we shall attempt in the next section.

4. Quantum Individuals and the Identity of Indiscernibles.

We begin by discussing the case of fermions. It has been claimed in the literature that the Pauli Exclusion Principle, prohibiting two fermionic particles from being in the same quantum state, is a clear vindication of PII.[9] What is being prohibited apparently is that the two fermions shall have both the same intrinsic state-independent properties of mass, spin, electric charge, etc. and the same state-dependent properties expressed by expectation values of all quantum-mechanical ~~observables~~ physical magnitudes. But look at the allowed state ~~VIII~~ (8). It is not true in such a state that each particle is present in a different state. Each particle clearly partakes of both the states $|a\rangle$ and $|a'\rangle$ in the superposition of product states expressed in ~~VIII~~ (8). So might it not appear that in the allowed state both particles also have the same state-dependent properties, which would contradict PII.

Let us formulate these state-dependent properties in terms of physical magnitudes such as Q_1 and Q_2 pertaining to each particle separately as discussed in the preceding section. On orthodox interpretations of quantum mechanics the properties Q_1 and Q_2 must be interpreted not as possessed values but as propensities to yield specified ~~measurement~~ 'actualization' results in accordance with the familiar statistical algorithm for computing the associated probabilities.

Insertion *

and remembering $\sum_n |q^n\rangle\langle q^n| = I$,

$\langle a^n|a^s\rangle = 0$ and $\langle a^n|a^n\rangle = \langle a^s|a^s\rangle = 1$

Denoting the fermion state (8) ~~VIII~~ by $|\Psi\rangle$ we shall be interested in ~~computing~~ ~~evaluating~~ comparing both monadic properties of the form $\mathrm{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha)$ and $\mathrm{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha)$

~~but also~~ and also relational properties of the form $\mathrm{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \,|\, Q_2 = q^\beta)$ and $\mathrm{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha \,|\, Q_1 = q^\beta)$

These quantities are ~~very~~ easily computed from

$$\mathrm{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha \,\&\, Q_2 = q^\beta)$$

$$= \left| \langle q^\alpha | \langle q^\beta | (|\Psi\rangle) \right|^2$$

$$\cancel{= \tfrac{1}{2} |\langle q^\alpha | a^r\rangle|^2 + \tfrac{1}{2} |\langle q}$$

$$= \tfrac{1}{2} |\langle q^\alpha | a^r\rangle|^2 \cdot |\langle q^\beta | a^s\rangle|^2$$

$$+ \tfrac{1}{2} |\langle q^\alpha | a^s\rangle|^2 \cdot |\langle q^\beta | a^r\rangle|^2$$

$$- \mathrm{Re}\, \langle a^r | q^\alpha\rangle\langle q^\alpha | a^s\rangle\langle a^s | q^\beta\rangle\langle q^\beta | a^r\rangle \quad \text{--- (10)}$$

Summing this result over ~~q^α or q^β~~ α or β to obtain the marginal probabilities [marginals] ~~and remembering~~ ~~$\sum_\alpha |q^\alpha\rangle\langle q^\alpha| = \sum_\beta |q^\beta\rangle\langle q^\beta| = \sum_n |q_n\rangle\langle q_n| = 1$ and $\langle a^r | a^s\rangle = 0$~~, yields immediately ~~the equality~~

$$\mathrm{Prob}^{|\Psi\rangle}(Q_1 = q^\alpha) = \mathrm{Prob}^{|\Psi\rangle}(Q_2 = q^\alpha)$$

$$= \tfrac{1}{2} |\langle q^\alpha | a^r\rangle|^2 + \tfrac{1}{2} |\langle q^\alpha | a^s\rangle|^2 \quad \text{--- (11)}$$

Similarly we find

$$\mathrm{Prob}^{|2I\rangle}(Q_1 = q^\alpha \,|\, Q_2 = q^\beta)$$

$$= \mathrm{Prob}^{|2I\rangle}(Q_2 = q^\alpha \,|\, Q_1 = q^\beta)$$

$$= \Big\{ |\langle q^\alpha | a^r\rangle|^2 \cdot |\langle q^\beta | a^s\rangle|^2 + |\langle q^\alpha | a^s\rangle|^2 \cdot |\langle q^\beta | a^r\rangle|^2 - 2\,\mathrm{Re}\,\langle a^r | q^\alpha\rangle\langle q^\alpha | a^s\rangle\langle a^s | q^\beta\rangle\langle q^\beta | a^r\rangle \Big\} \Big/ |\langle q^\beta | a^r\rangle|^2 + |\langle q^\beta | a^s\rangle|^2 \quad \text{--- (12)}$$

The significance of (11) and (12) is that the two fermions in the state (8) do in fact have the same monadic properties and the same relational properties one to another, so the weakest form of PII' which we can formulate ~~is in fact violated~~ which involves both monadic properties and relational properties, is violated.

There are a number of comments we want to make concerning this conclusion and the way it was derived.

(1) In classical physics the state-dependent properties of a particle are completely specified by the maximally specific state

description (location in phase space).
Hence we can replace the questions, "Do diamonds [illegible] have the same state-dependent properties?" with the question, "Do the two particles have the same maximally specific state description?"

If we try the same move on quantum mechanics we run into the problem that for a so-called 'entangled' state such as (8) there are no pure states which can be ascribed to the separate particles. (If there were such states the state of the combined system would be the tensor product of the states in question but (8) is not of the form of a tensor product — it's a superposition of tensor products.)

Now pure states in QM play the role of ~~correspond to the~~ maximally specific states, so if we identified the relevant properties of the two particles with the pure states they are in we would have to conclude that there is no answer to the question, "Do the they have the same properties?"

A corollary of this result is that insofar as we can speak of states for the separate particles at all we must speak of mixed states.[10] Indeed the relevant mixed states are the same for the two particles[11] — equiprobable mixtures of the states $|+^2\rangle$ and $|9^5\rangle$. This is of course the essential content of the result (10) for the marginal probabilities (7).

distributions for Q_1 and Q_2. But our analysis has gone beyond that involving the (improper) mixed states of the separate particles, by considering also the ~~new~~ 'relational' conditional probabilities given in (12).

(2) There is another sort of relational property we might consider expressed by conforming

$$\mathrm{Prob}\left(Q_2 = q^\alpha \mid Q_2 = q^\beta\right) = \delta_{\alpha\beta} \qquad (13)$$

with $\mathrm{Prob}(Q_1 = q^\alpha \mid Q_2 = q^\beta)$ given by (12)

These relational properties of particle 1 to itself as compared with relations of particle 1 to particle 2 we reject as vindication of PII, for the same argument as we discussed in section 1. ~~[This property does already presuppose individuation of particle 1, and hence cannot be used to account for this individuation via PII.~~
~~In the case of the two conditional probabilities calculated in (12) this argument does not apply, since we are evaluating relations that exist between one particle and the other, and asking the question, does particle 1 exhibit the same to the other particle~~ ~~each particle exhibit the same relation to the other particle.~~
~~This does not presuppose]~~ for ruling out illegitimate trivialisation of PII. The purported vindication of PII again depends

or regarding (13) as a monadic property ~~of particle~~ of particle 2 whereas it is in fact a relational property of particle 2 to itself which is also true as a relation of particle 1 to itself.

(3) If we write $\alpha = \beta$ in equation (12) then we indeed find for fermions the result

$$Prob\,(Q_1 = q^\alpha \mid Q_2 = q^\alpha) = 0$$

This shows that if ~~a measurement~~ ~~result~~ an actualization of Q_2 gives a certain value then there is zero probability that a ~~measurement~~ concurrent actualization of Q_1 will ~~be~~ yield the ~~same~~ same value. This is the real expression of the Exclusion Principle but has no bearing on PII if we adhere to the orthodox view that ~~measurement~~ ~~results~~ actualizations do not correspond to antecedently existing possessed values.

(4) This brings us to our final comment. On hidden-variable reconstructions of QM, the circumstance described in point 3 above, would lead us to regard PII as vindicated for fermions (assuming ~~not~~ ~~faithful~~ ~~of faithful~~ ~~measurement~~ that ~~measurement~~ actualization results merely reveal pre-existing values).

New para

We now turn to the case of bosons.

It is often assumed that purported violation of PII depends on consideration of states such as (3) & (6) where both particles ~~are indeed~~ can indeed be attributed the same pure state[13].

Denoting the state (3) by $|\Phi\rangle$ for example ~~yields the~~ we can easily obtain the following results corresponding to (11) and (12):

$$\text{Prob}^{|\Phi\rangle}(Q_1 = q^\alpha) = \text{Prob}^{|\Phi\rangle}(Q_2 = q^\alpha)$$
$$= |\langle q^\alpha|\alpha^2\rangle|^2 \quad \text{--- } (11')$$

and
$$\text{Prob}^{|\Phi\rangle}(Q_1 = q^\alpha | Q_2 = q^\beta) = \text{Prob}^{|\Phi\rangle}(Q_2 = q^\alpha | Q_1 = q^\beta)$$
$$= |\langle q^\alpha|\alpha^2\rangle|^2 \quad \text{--- } (12')$$

So, as we might expect, both monadic and ~~relational~~ relational properties are the same for the two particles.

But it should be noted that this conclusion is also true for the state (7) where two different states are ~~involving~~ involved. In this case the results (11) and (12) apply with the minus sign in front of the 'interference' term in (12) replaced by a plus sign.

Finally we make a brief comment on the case of paraparticles.

Here there do exist states for which the monadic properties of all the separate particles are not the same, but equally there

to be made for transpositions, since any permutation can be represented as a product of transpositions, so even permutations are always associated with the plus sign, the distinction between bosons and fermions only arising for odd permutations).

But and Messiah ~~and~~ Greenberg (1964) pointed out that (1) should, on a more profound analysis, be interpreted not as a restriction on states, but as a restriction on the possible observables for the N-particle system. On this view (1) can easily be shown to imply $P^{-1}QP = Q$ or $QP = PQ$, so any permitted Q must commute with any permutation P. This in turn implies that Q must be a symmetric function of the particle labels. The label permutations provide effectively a set of non-Abelian superselecting operators which can be used to resolve the state space into non-cohering sectors associated with irreducible representations of the symmetric group S_N.

For two particles there are only two irreducible representations of S_2, ~~the symmetric~~ provided by states which are symmetric or antisymmetric in the particle labels. So we are back to the boson and fermion possibilities but, with more than two particles higher-dimensional representations of the symmetric group

~~do exist~~ are possible paraparticle states for which SII is violated in the same way as for bosons and fermions.)

As an example consider the following normalized[14] state for three parabosons of order 2[14]:

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\left(|a^r\rangle|a^r\rangle|a^s\rangle - |a^s\rangle|a^r\rangle|a^r\rangle\right) \quad (\cdot\,14)$$

where $|a^r\rangle$ and $|a^s\rangle$ are two distinct one-particle states and triple tensor products are written in the sequence of particle labels 1, 2 and 3.

Denoting $Q \otimes I \otimes I$ by Q_1, $I \otimes Q \otimes I$ by Q_2 and $I \otimes I \otimes Q_3$ by Q_3, we calculate for the triple joint distribution

$$\begin{aligned}
\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha, Q_2 = q^\beta, Q_3 = q^\delta) &= \left|\langle q^\alpha| \otimes \langle q^\beta| \otimes \langle q^\delta| \left(|\Psi'\rangle\right)\right|^2 \\
&= \frac{1}{2}\Big[|\langle q^\alpha|a^r\rangle|^2 \cdot |\langle q^\beta|a^r\rangle|^2 \cdot |\langle q^\delta|a^s\rangle|^2 \\
&\quad + |\langle q^\alpha|a^s\rangle|^2 \cdot |\langle q^\beta|a^r\rangle|^2 \cdot |\langle q^\delta|a^r\rangle|^2 \\
&\quad - 2\,\mathrm{Re}\,\langle a^r|q^\alpha\rangle\langle q^\alpha|a^s\rangle \cdot \langle a^r|q^\beta\rangle\langle q^\beta|a^r\rangle \cdot \langle a^s|q^\delta\rangle\langle q^\delta|a^r\rangle\Big]
\end{aligned} \quad --(15)$$

From (15) we find immediately the marginal distributions

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha) = \mathrm{Prob}^{|\Psi'\rangle}(Q_3 = q^\alpha)$$
$$= \tfrac{1}{2}\left(|\langle q^\alpha | a^r\rangle|^2 + |\langle q^\alpha | a^s\rangle|^2\right) \quad \text{--- (16)}$$

while

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_2 = q^\alpha) = |\langle q^\alpha | a^r\rangle|^2 \quad \text{--- (17)}$$

Thus particles 1 and 3 have the same monadic properties expressed by the marginal distributions but these differ from the monadic properties of particle 2.
Let us now show that particles 1 and 3 also have the same relational properties with respect to both the remaining particles.

We ~~can~~ easily find that ~~show that~~

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha | Q_3 = q^\gamma) = \mathrm{Prob}^{|\Psi'\rangle}(Q_3 = q^\alpha | Q_1 = q^\gamma)$$
$$= \Big[\, |\langle q^\alpha|a^r\rangle|^2 \cdot |\langle q^\gamma|a^s\rangle|^2 + |\langle q^\alpha|a^s\rangle|^2 \cdot |\langle q^\gamma|a^r\rangle|^2 - 2\,\mathrm{Re}\,\langle a^r|q^\alpha\rangle\langle q^\alpha|a^s\rangle\langle a^s|q^\gamma\rangle\langle q^\gamma|a^r\rangle \Big] \Big/ \Big[\, |\langle q^\alpha|a^r\rangle|^2 + |\langle q^\alpha|a^s\rangle|^2 \Big]$$

(18)

Furthermore

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha \mid Q_2 = q^\beta)$$
$$= \mathrm{Prob}^{|\Psi'\rangle}(Q_3 = q^\alpha \mid Q_2 = q^\beta)$$
$$= \tfrac{1}{2}\left[|\langle q^\alpha | a^r\rangle|^2 + |\langle q^\alpha | a^s\rangle|^2 \right]$$

(19)

and finally

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha \mid Q_2 = q^\beta \,\&\, Q_3 = q^\gamma)$$
$$= \mathrm{Prob}^{|\Psi'\rangle}(Q_3 = q^\alpha \mid Q_2 = q^\beta \,\&\, Q_1 = q^\gamma)$$
$$= \Big[|\langle q^\alpha | a^r\rangle|^2 \cdot |\langle q^\beta | a^s\rangle|^2 \cdot |\langle q^\gamma | a^r\rangle|^2 + |\langle q^\alpha | a^s\rangle|^2 \cdot |\langle q^\beta | a^r\rangle|^2 \cdot |\langle q^\gamma | a^r\rangle|^2$$
$$- 2\,\mathrm{Re}\, \langle a^r | q^\alpha\rangle\langle q^\alpha | a^s\rangle\langle a^s | q^\beta\rangle\langle q^\beta | a^r\rangle \langle a^r | q^\gamma\rangle\langle q^\gamma | a^r\rangle \Big]$$
$$\Big/\ |\langle q^\beta | a^r\rangle|^2 \left[|\langle q^\gamma | a^r\rangle|^2 + |\langle q^\gamma | a^s\rangle|^2 \right]$$

(20)

These results show that PII is violated for particles 1 and 3 in the state $|\Psi'\rangle$, even in its weakest form, that includes all the relevant relational properties.

so this is a case where PII is violated for particles 1 and 2.

5 Conclusion

There are two main conclusions of this paper. Firstly that indistinguishable particles in QM can be treated as individuals, but secondly, if they are so treated, then, on the most plausible reading of 'what' constitutes a property of a quantal particle, then even the weakest form of PII including both monadic and relational properties, is violated both for bosons and fermions and indeed for higher-order paraparticles.

It should be noted that if quantal particles are individuals then their individuality must be conferred by TI. STC is not in general available in QM, since particles do not move in well-defined trajectories so the question of spatio-temporal continuity of ~~path~~ trajectory does not arise. The only exception to this is where the one-particle states involve well-defined wave packets, which diffuse sufficiently slowly over time, as would be possible for the classical limit of sufficiently massive particles.

But it is clear that in the case of macroscopic bodies, where STC can be used to ~~label the~~ ~~indiv~~ label the bodies,

Question

* assuming them to be maximally specific.

the STC criterion actually conflicts with the TI understanding of the ~~lore for~~ elementary particles comprising the body.
To be strict every electron, for example, partakes of the state of every other electron in the universe according to the anti-symmetrization requirement!
But notice, that under conditions where the 'interference' term in (10) can be neglected, ~~that~~ then the state $|\Psi\rangle$ behaves like a *proper* mixture of states in which particle 1 is in state $|a^r\rangle$ and particle 2 in state $|a^s\rangle$ and the permuted state in which particle 1 is in state $|a^s\rangle$ and particle 2 in state $|a^r\rangle$, with equiprobable weights for the two different states in the mixture. ~~That~~ So under these conditions the state (8) behaves like an equiprobable mixture of states (3) and (4). In other words, when 'interference' can be neglected, we recover the ~~state~~ same possibilities for states as in classical physics, ~~since~~ though states (1) and (2) would anyway be eliminated by IA Invariance.
But, of course, ontologically speaking, 'interference' is never strictly absent. That, after all, is what constitutes the 'problem of measurement' in QM, so the involvement of every electron with the state of every other electron in the universe, although negligible for ~~many~~ practical purposes, remains an ontological commitment of QM under the interpretation where the particles are treated as individuals.

If this sounds too bizarre to be acceptable, it provides another argument for preferring the treatment of indistinguishable particles along the lines provided by quantum field theory.

In this paper we have been concerned with conceptual possibilities, rather than what it is most reasonable to believe about the ontological status of elementary particles.

~~Acknowledgement~~ ~~One of us~~

Insertion to Note 2.

* The *locus classicus* is generally held to be Leibniz (1961) Book II chapter XXVII §§1–3. For an ~~influ~~ recent influential critical discussion see in particular Shigure (1976).

Note ~~11~~ 12 It should be stressed that these relational properties expressed by the conditional probabilities in no way supervene on the monadic properties expressed by the marginal distributions. In the terminology of Teller (1986) they are ~~xxxxx~~ inherent relations.

Note 1 There is much discussion in the literature as to whether a clear-cut distinction can be made between relational and monadic predicates. For a useful comment see Hoy [1984].

Notes ⇒ Ph.D Acknowledgment - See p. 28.

2 ~~1~~. There is no real consensus about Leibniz's own views on the status of PII. ~~Frankfurt [1972] is an authoritative and collection of critical essays that deal with this aspect (among many others) of Leibniz's philosophy. See in particular the contribution by Ishiguro entitled~~ *

~~2~~3 This terminology is due to Post [1963].

5 ~~4 3~~. This interpretation of PII is mooted in Lucas [1984] p. 131.

6 ~~8 4~~. The justification for assuming that the Hamiltonian observable must be a symmetric function of the particle labels will become apparent in Section 3.

4 ~~5~~. In the physics literature such particles are often referred as 'identical'. In our terminology this would mean they were one and the same particle! We shall use the term 'indistinguishable' ~~where~~ in place of the physicist's 'identical'.

7 ~~6~~. See Redhead [1983] and [1986] for further discussion of the QFT approach.

~~7.~~ 8. We refer to 'actualizations' rather 'measurement results', to emphasise the point that they are not observable. They will, however, be produced by measurement interactions, but not in a way which makes them identifiable as contrasted with their particle label-permuted variants.

~~8~~ 9 See ~~Margenau [1944]~~ and [1950] ~~, and~~ Shadmi [1978]

~~9~~ ~~12~~ 9 10 See D'Espagnat [1976] pp. 58–61 for a discussion of mixed states arising in this way. He calls them 'improper' mixtures.

~~12~~ ~~11~~ ~~10~~ 13 See, for example, the discussion of FDI for bosons given by Cortes [1976], Barnette [1978], Ginsburg [1981] and Teller [1983].

~~12~~ ~~13~~ 14 Compare ~~Holt~~ Hartle and Taylor [1969] for details of how to construct permuted states. The state (14) is obtained from their (2.5) by identifying two ~~one~~ of the one-particle states.

~~10~~ ~~9~~ 11 second A similar point is made in Van Fraassen [1984]. See also Margenau [1944] and [1950].

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are always ~~other~~ possible paraparticle states where PII ~~is~~ violated, in the same way as for bosons and fermions.

As an example consider the following normalized state for three parabosons of order 2 [12]

$$|\Psi'\rangle = \frac{1}{\sqrt{2}}\left(|a^r\rangle\otimes|a^s\rangle\otimes|a^r\rangle - |a^s\rangle\otimes|a^r\rangle\otimes|a^r\rangle\right) \quad (14)$$

where $|a^r\rangle$ and $|a^s\rangle$ ~~and $|a^t\rangle$~~ are ~~three~~ two distinct one-particle states and triple tensor products are written in the sequence of particle labels 1, 2 and 3.

Denoting $Q\otimes I\otimes I$ by Q_1 ~~and~~ $I\otimes Q\otimes I$ by Q_2, and $I\otimes I\otimes Q$ by Q_3 we easily find

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_1 = q^\alpha) = \mathrm{Prob}^{|\Psi'\rangle}(Q_2 = q^\alpha) = \frac{1}{2}\left(|\langle q^\alpha|a^r\rangle|^2 + |\langle q^\alpha|a^s\rangle|^2\right) \quad (15)$$

while

$$\mathrm{Prob}^{|\Psi'\rangle}(Q_3 = q^\alpha) = |\langle q^\alpha|a^r\rangle|^2 \quad (16)$$

~~However if we make r = s, then the two marginal probabilities do become equal, and we can easily verify that the relational properties of particles with respect to both the remaining particles are also the same,~~